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# DAPS

# Deflection Analysis of Pavement Structures

## Back-calculation of Layer Stiffness from Deflection Data using the Singular Value Decomposition Technique

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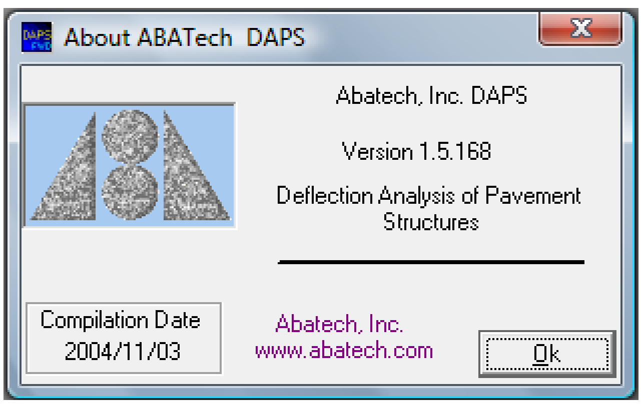
# Introduction

Pavement structures provide a vital part of our transportation system. Efficient maintenance is necessary in order to achieve maximum cost benefit from the huge investment made by transportation agencies over the years. As part of maintenance planning and analysis, agencies have been making use of deflection testing equipment such as the Falling Weight Deflectometer. This type of equipment yields information regarding the structural performance in terms of deflections that are used to calculate layer stiffness modulus. DAPS – Deflection Analysis of Pavement Structures – is a rapid, accurate and reliable method for performing back-calculation of deflection results.

The software is written using modern windows programming languages – with this version being developed for the Windows 98, NT, 2000, XP, Vista operating systems.



# Theoretical Background

As deflection data is increasingly used to accurately define pavement response to loading the need to develop robust back-calculation procedures has become greater. With the speed of computers increasing more mathematically intensive procedures can be used for developing solutions of large data sets. Recently, a Deflection Analysis of Pavement Structures (DAPS) software has been developed, which enables the rapid calculation of layer stiffness modulus using a singular decomposition technique.

The back analysis algorithm solves for both a two layer elastic system and the thickness of subgrade, or a three layer elastic system and the thickness of subgrade. A least square solution process is applied, employing all the measured deflections as parameters characterizing the bowl.

A rigid base beneath the subgrade is assumed ('bedrock'). This is an accepted method, to some extent, to allow for known effects of non-linearity within the subgrade soil. The rigid base depth is used as an unknown to be solved for, along with the layer modulus.

Seed values for the AC and subgrade stiffnesses are obtained from equations published by Thompson (1989), using deflections d0 to d3. These are used to generate trail values of the parameters characterizing the bowl (i.e. the deflections). If a granular base is assumed to be present, the granular base resilient modulus seed value is estimated by empirical relations (Thompson, 1982). If a three-layer system without granular base is to be solved, the subgrade E estimate can be used for the base layer also. An arbitrary fixed initial trail value of subgrade thickness is employed, viz. 7m.

As described so far, it is evident that there are 7 known parameters, and either 3 or 4 unknowns, viz. 2 E's and 1T, or 3E's and 1T. Since there are more parameters than unknowns, an over-determined set of simultaneous equations can be set up relating changes in the unknowns to changes in the deflections by means of a matrix of partial derivatives, dpi/dUj, where p are the deflections and U are the unknowns (either E values or thickness value).

A least squares solution to these simultaneous equations is obtained by an iterative process using, at each iteration, a solution of the over-determined equation set by the Singular Value Decomposition technique (Press et al., 1986).

The difference between the computed deflections based on the initial unknown's estimates (seed values), and the measured deflections are hence minimized by the following procedure for updating the unknowns:

$P_k a_k = r_k$

where:

> $P_k$ = the $k_{th}$ iteration of the matrix of partial derivatives $dp_i/dU_j$ of the parameters p1, I=1 to 7, with respect to the 'unknown' layer modulus and thickness $U_j$, j=1 to 3 or 1 to 4.
>
> $a_k$ = the $k_{th}$ difference vector, which is the differences $U_{j,k+l} - U_{j,k}$ between the modulus/thickness used in the $P_k$ matrix and the new modulus/thickness $U_{j,k+l}$ to be used in the (k+1)th iteration.
>
> $r_k$ = the residual vector of differences between the most recently computed parameters and the parameters represented by the measured deflections.

In the above equations, the partial derivatives comprising the P matrix are estimated numerically, by Elastic Layer analysis. At present, no limits are applied to E values generated by the minimization procedure.

This back-calculation procedure is considered to be suitable for:

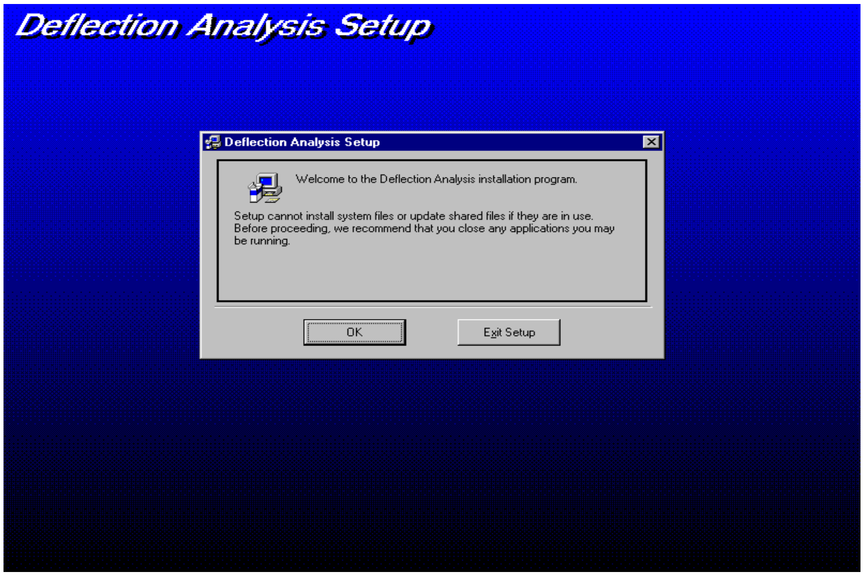
- Full depth asphalt concrete (AC)
- Conventional (AC surface plus granular base)
- AC + high-strength stability base (HSSB)
- PCC over granular base

# Program Installation

The program is supplied on a single CD-ROM or via the Internet. The user runs the setup.exe program using normal windows execution (e.g. from run menu a:\setup.exe) and then follows the instructions given by the software.

The setup creates a default directory of c:\daps and c:\daps\data1. The data directory is the default data directory for storing deflection data for analysis. The user can change this if necessary along with the program directory during the installation process.

Following installation it is recommended that the user restarts the computer to ensure that all setting changes have taken place.



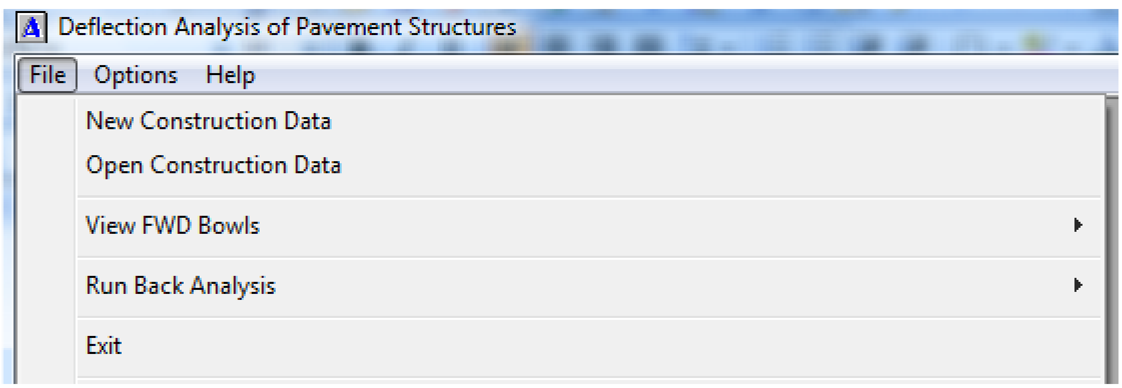
# Program Use

The user runs the program from the Start-Program menu or using other standard window methods and then starts by selecting the file menu as illustrated.

The file menu enables the user to choose an area to work in. The choices are illustrated and are as follows:

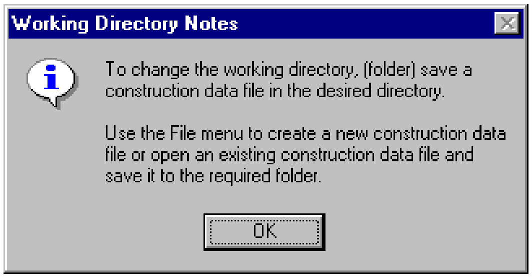
- New Construction Data
- Open Construction Data
- View FWD Bowls
- Run Back Analysis
- Exit

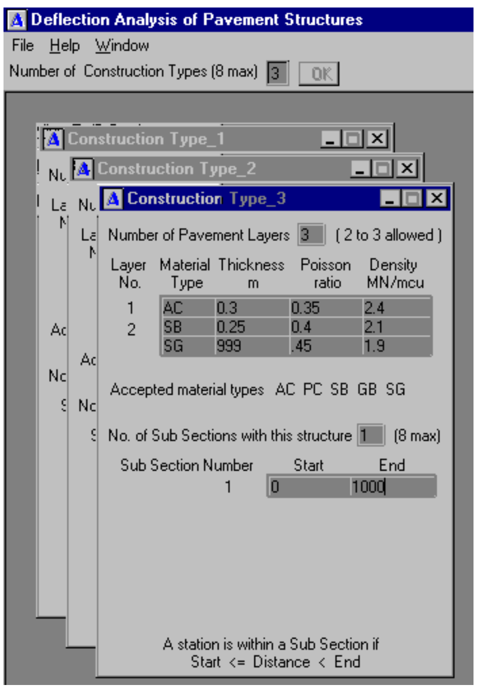
Each of these areas will be considered in detail in the following pages.



# Construction Data

Pavement construction information must be defined to enable back calculation. This is done via the creation of a construction information file. The file should have the same root name as the FWD data file and would be normally located in the default directory c:\daps\data unless otherwise specified by the user – see working directory notes below.




The pavement construction information can be defined for various lengths of the pavement by reference to the station numbers. In the example given, three construction types have been defined for the FWD data file. The user must define a length of pavement associated with each construction type.

The information stored will be as follows:

Material Types:

- AC – Asphalt Concrete
- PC – Portland Cement
- GB – Granular Base
- SB – Granular Sub-base
- SG – Subgrade

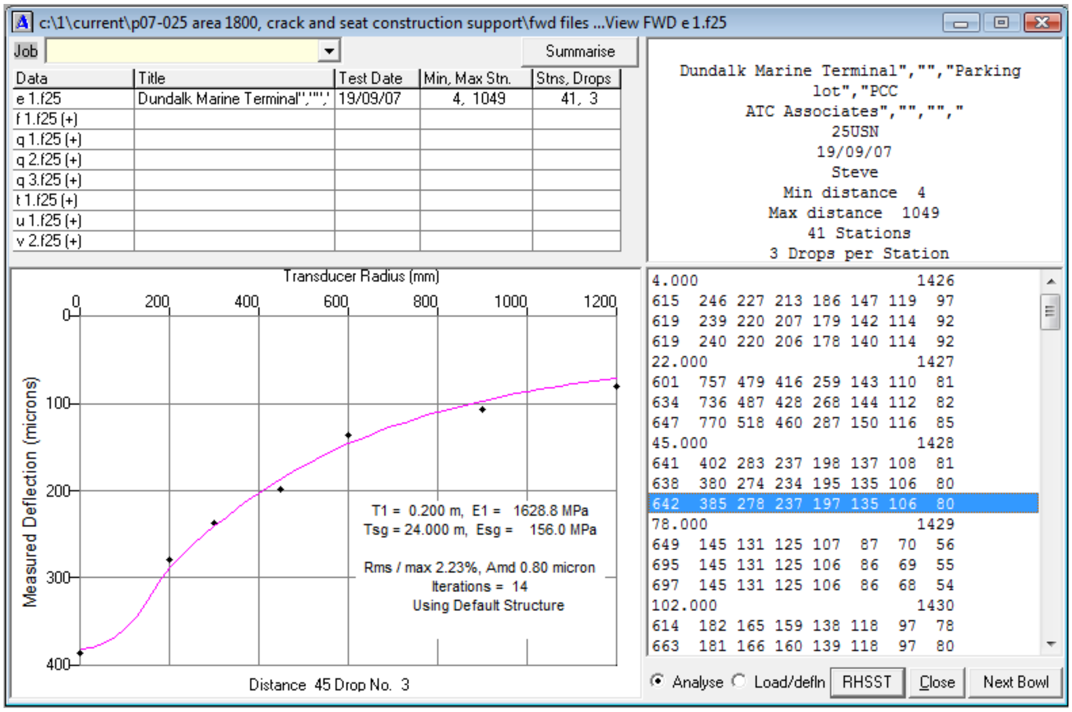
Poisson's ratio must be less that 0.5. Thickness and density inputs are in metric units.

When the user has completed the data entry in this area he uses the Save or Save As file commands to store the file.

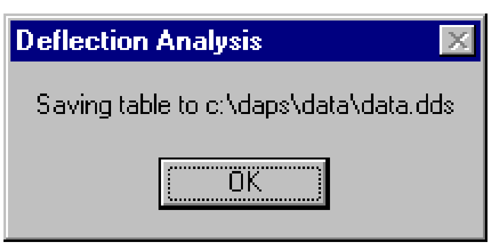
An alternative to starting a new construction information file is to use the data in an existing file by opening and then use the Save As command to a new file name after completion/modification of data.

# View FWD Bowls

When selecting the menu option View FWD Bowls the user is presented with the form as illustrated below.




Data files in the working directory are listed under the data item in the grid. The program recognizes most common FWD file formats, including one EXCEL spreadsheet format. Initially, all other information items are blank. Hitting the Summarize button fills in this information. This summary can be saved to the hard disk. The data is stored in the file data.dds and is illustrated below.



This process determines the number of deflection bowls in the data file and allows the user to review the distance/station information to be reviewed with ease. Often the user will want to summarize data before setting construction information for a file:

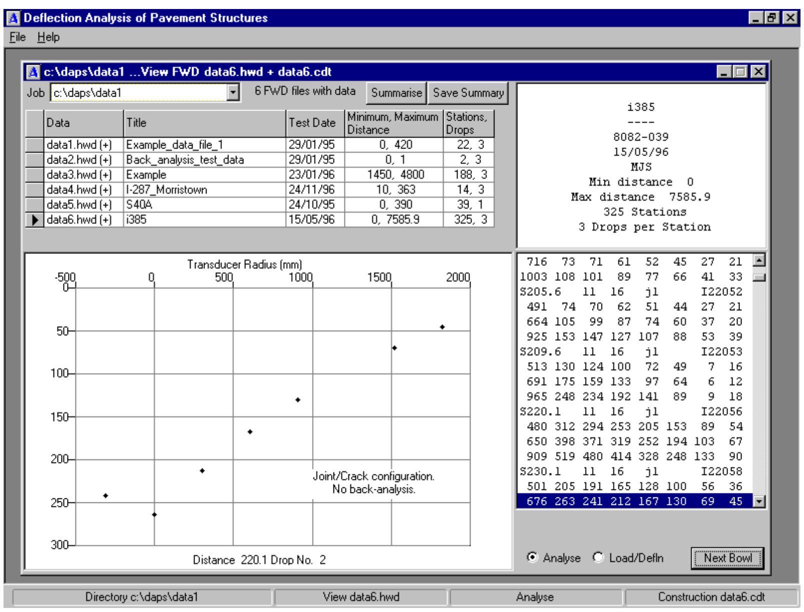
```
FWD Raw Data. Files in Directory c:\daps\data, on 08-31-98, at 18:36:15

File      Title                            Date of  Min     Max      No.of  No.of
Name                                       Testing  Dist.   Dist.    Stns.  Drops

data1     Example_data_file_1              29/01/95  0,   420        22,  3
data2     Back_analysis_test_data          29/01/95  0,   1          2,   3
data3     Example                          23/01/96  1450,  4800     188,  3
data4     I-287_Morristown                 24/11/96  10,  363        14,  3
data5     i385                             15/05/96  0,   7585.9     325,  3
data6     i385                             15/05/96  0,   7585.9     325,  3
```

Summarized data

**<u>The plus symbol next to the data filename indicates that a CDT (construction data) file has been completed</u>**, which is necessary to perform the back-calculation. A box towards the upper right corner of the screen contains general information in the file. Title, test date, distance information, number of stations and drops per station are displayed in the grid.

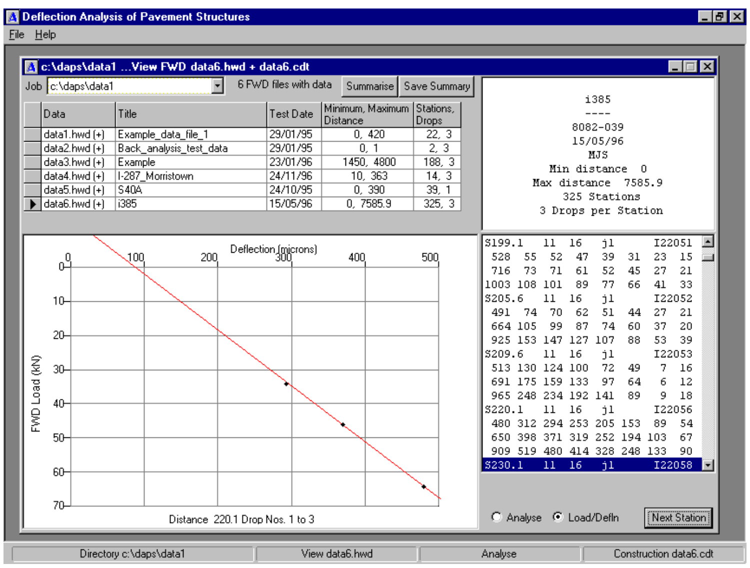



Single data points can be analyzed by interactively clicking on a deflection bowl (providing a CDT file has been made). Information displayed on the deflection bowl graph includes the layer stiffness and the calculated stiffness for each pavement layer. The thickness of the soil layer is the calculated thickness to an assumed rock foundation. The statistic RMSerr/Max

(Root Mean Square error divided by Maximum Deflection) is also given. A RMSerr/Max of less than 4% is considered as a satisfactory deflection bowl match.

It is recommended that users use this feature to inspect the data files and determine which model most accurately fits the back-analysis.

In locations where tests have been carried out over joints/cracks a note is given on the screen stating that "No back-analysis" has been conducted. Often deflection tests are conducted on jointed pavements at several stress levels to determine the performance of the joint. By clicking on the "Load Defln" (Load versus Deflection) radio button it is possible to view a graph of the pavement deflection versus the stress level.



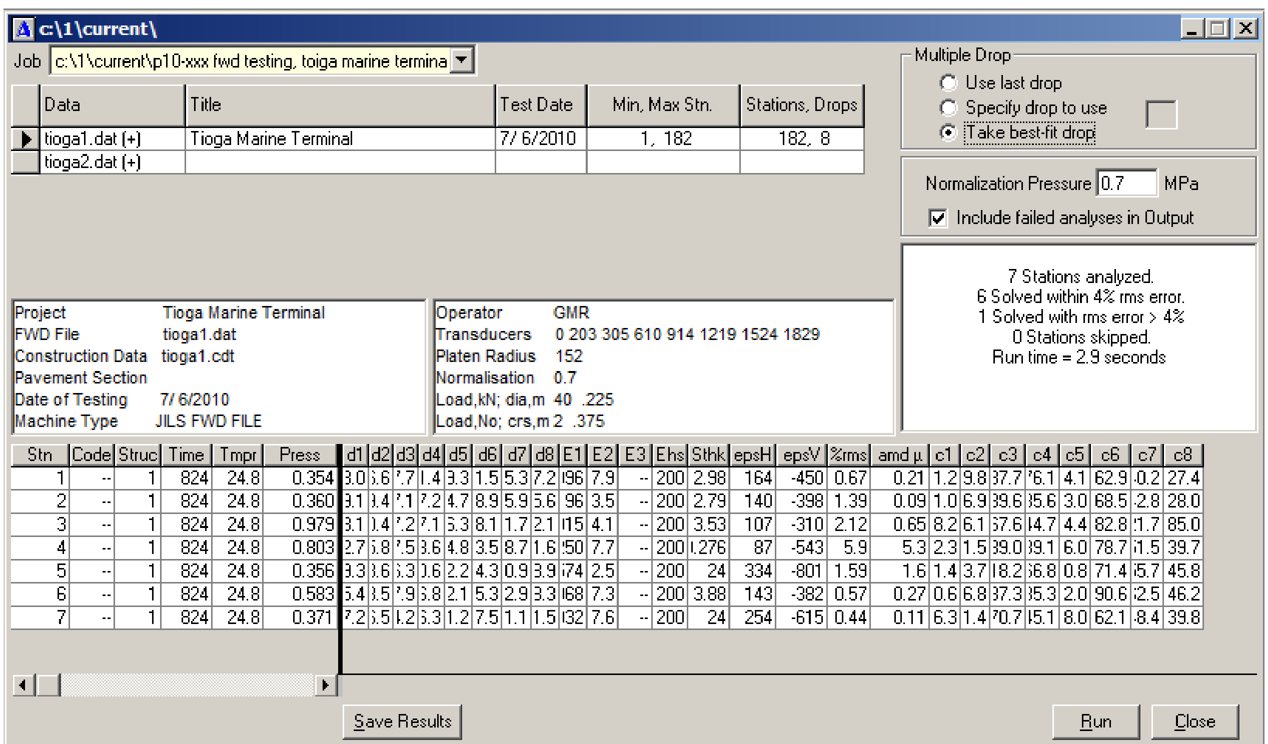
# Process FWD Data

A file which can be processed is indicated by a (+) next to the data file name. The file is selected and processed by selecting the "run" button. Information is updated as the analysis takes place in the display grid. The modulus of the layers are displayed (E1 to E3) along with the thickness of the soil layer to bedrock (Sthk), the calculated horizontal strain (tensile) at the underside of the bound layer (epsH) and the vertical strain at the top of the soil layer (epsV). In addition the %rms (Root Mean Square error divided by Maximum Deflection) is given in the last column. As discussed earlier a %ms error of less than 4% is considered as a satisfactory deflection bowl match. The summary window (middle right) indicates general statistical information on the analysis such as number of bowls successfully analyzed. The user can save the results by clicking on the Save Results button. In addition, the user is able to review the data and make any changes to the construction information, if required, and rerun the analysis.

The file, when saved, is stored in a BLS file that is formatted to enable easy import into a spreadsheet for further manipulation and/or analysis. The order of the data in the bls file matches the columns given in the screen below and are as follows:

| Column # | Description |
|---|---|
| Stn | Station number |
| Code | If entered a code is shown |
| Struc | A code number for a structure |
| Time | If entered a time will be displayed |
| Press | Pressure on loading platen |
| d1 | Deflection of 1$^{st}$ sensor |
| d2 | Deflection of 2$^{nd}$ sensor |
| d3 | Deflection of 3$^{rd}$ sensor |
| d4 | Deflection of 4$^{th}$ sensor |
| d5 | Deflection of 5$^{th}$ sensor |
| d6 | Deflection of 6$^{th}$ sensor |
| d7 | Deflection of 7$^{th}$ sensor |
| d8 | Deflection of 8$^{th}$ sensor |
| E1 | Stiffness modulus of top layer |

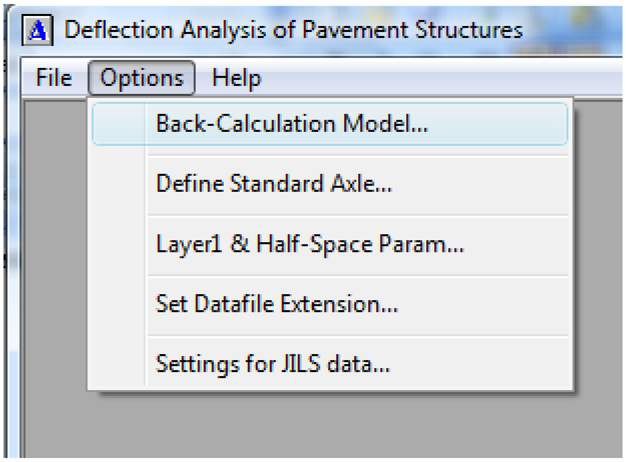
| | |
|---|---|
| E2 | Stiffness modulus of second layer |
| E3 | Stiffness modulus of third layer |
| Ehs | Stiffness modulus of half space |
| Stkh | Thickness calculated to bed-rock |
| epsH | Calculated horizontal strain at under side of lowest asphalt layer |
| epsV | Calculated vertical strain on top of half space (assumed soil) |
| %rms | Root mean square error of bowl fit compared to measured deflections |
| amd µ | Absolute micron deviation of calculated bowl from fitted bowl |
| c1 | Calculated 1st deflection using solved values |
| c2 | Calculated 2nd deflection using solved values |
| c3 | Calculated 3rd deflection using solved values |
| c4 | Calculated 4th deflection using solved values |
| c5 | Calculated 5th deflection using solved values |
| c6 | Calculated 6th deflection using solved values |
| c7 | Calculated 7th deflection using solved values |
| c8 | Calculated 8th deflection using solved values |



The other options on this window allow the user to use the last drop for the analysis, specify which drop to use or to use the best fit drop. If the best fit drop is selected then the software will attempt an analysis with each deflection bowl at a given location but only save the data with the lowest %rms error. The user may also specify a normalization pressure. This adjusts all the output to a constant applied stress rather that the deviating stress that is typically found in FWD deflection data. In addition, failed output can also be included. This is a useful feature since it enables an engineer to see which deflection bowls could not be analyzed. The engineer may choose other methods to estimate a stiffness modulus for these locations.

# Options

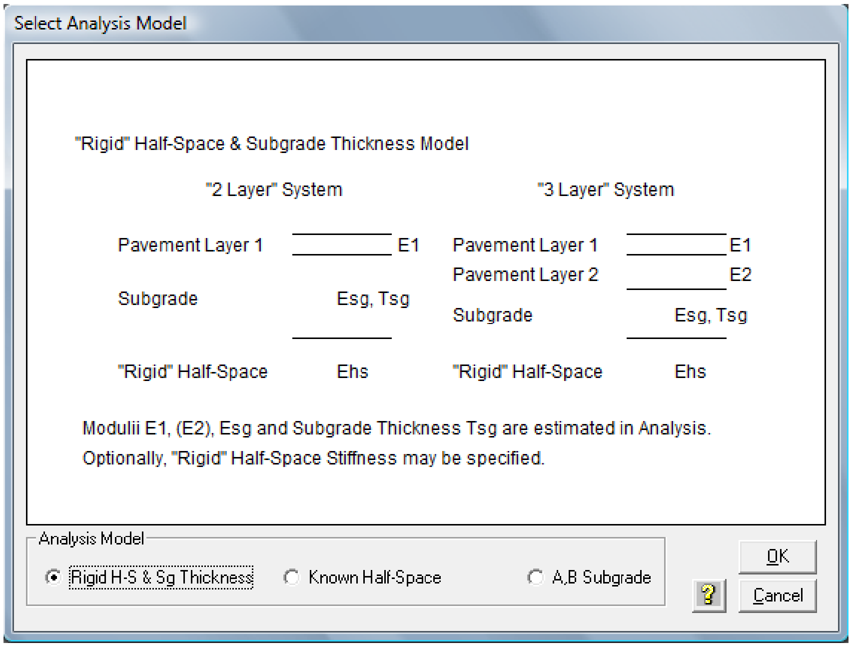
Several user options are provided as discussed below:

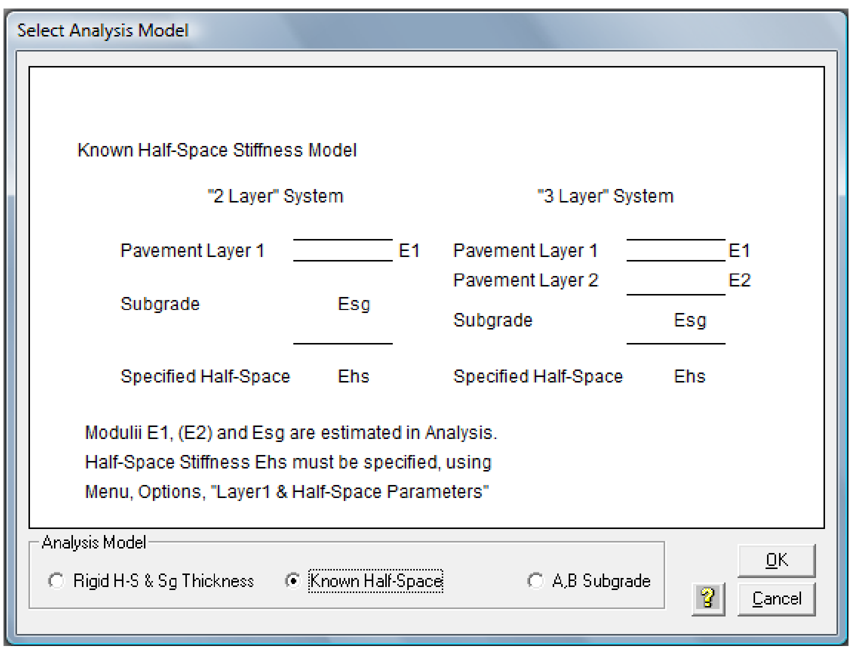


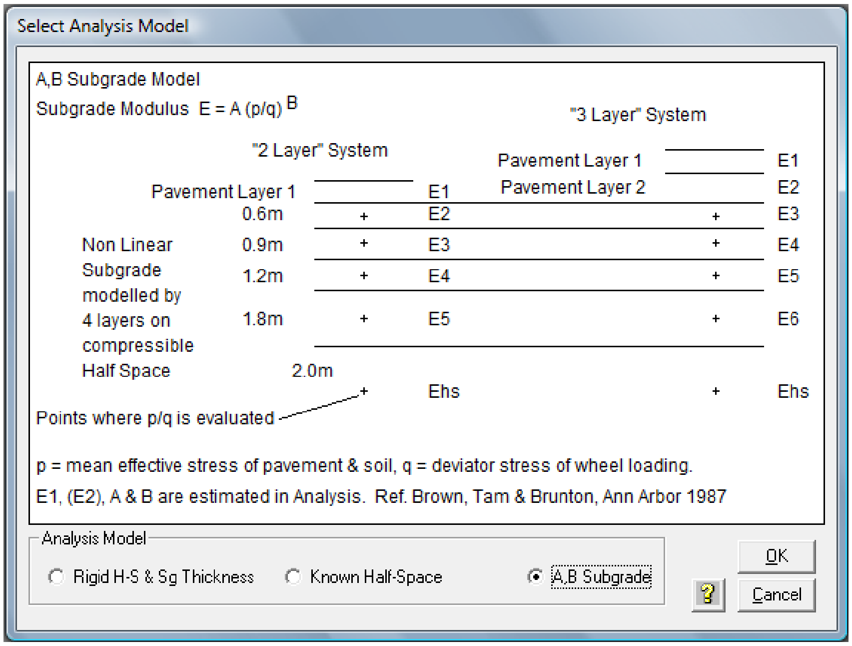

## Back-calculation model

The use is allowed to choose one of a three soil models to represent the pavement analysis. The selection can be made by selecting the "radio button" on this option. In addition, the user can toggle between results of the different soil models in the "View FWD Bowls" using the button on the lower right side of the screen. The models enable a greater understanding of the soil foundation than many other back-analysis software programs. The depth to bedrock calculation gives a similar result to that produced by MODULUS (developed by Texas Transportation Institute) whereas the stress dependent soil model is comparable to that in the PADAL software produced by the University of Nottingham. The stiffness adopted for the bed-rock will depend upon the local geology. For example, testing in the Bahamas, we have noted that a value of 1,000 MPa works well for a weak soft limestone. In other parts of the world where hard rock is found we have used a significantly higher value. If testing in large depths of granular material (for example – glacial deposits) we have adopted a value more consistent with the a high granular stiffness (say 300 to 500 MPa) in the analysis.

The third model – with a bed rock of know stiffness was developed for back-analysis of structures with deep alluvial layers in the New York area and may be suitable for coastal regions with these types of deposits. In the analysis with this type of feature we would recommend a low value of stiffness is adopted for the deep layers – with this value being consistent with data obtain from geo-technical investigations.

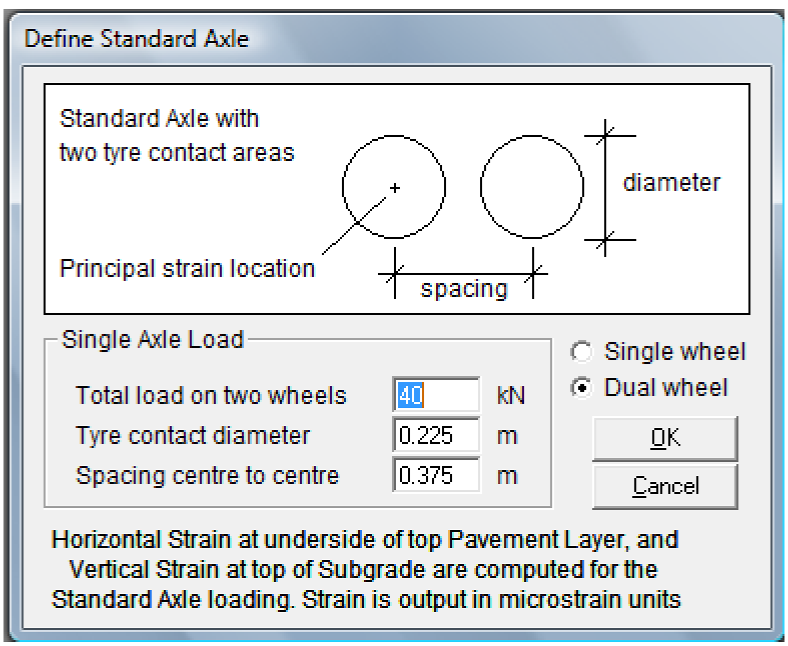




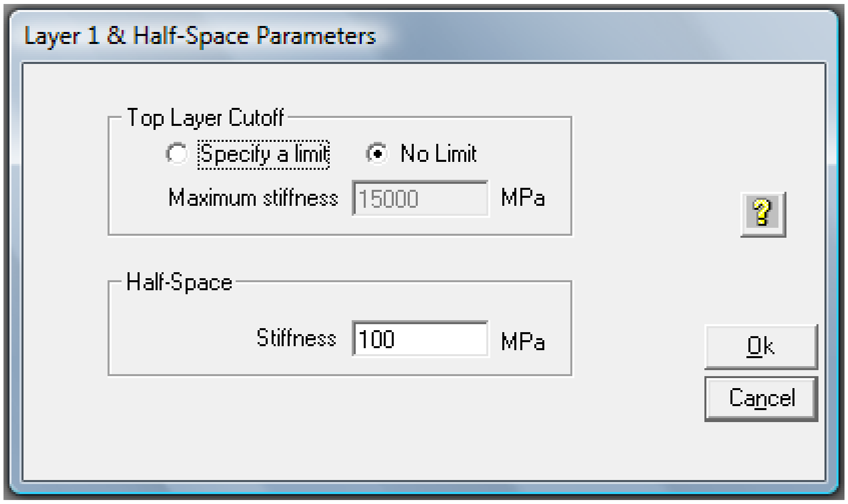

## Define standard axle

In the out-put file (*.bls) a calculation is provided of the strain in critical locations. The standard axle defined in this window is used to determine these strains.



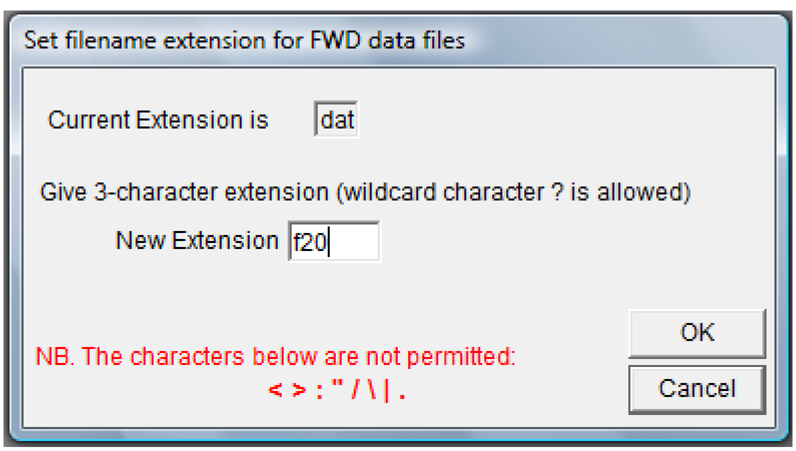
## Layer 1 & Half Space Parameters

The upper layer modulus can have a cut-off value selected. This is useful if analyzing very thin pavement layers. In addition, it is possible to set a half-space stiffness (associated with the rock at depth).



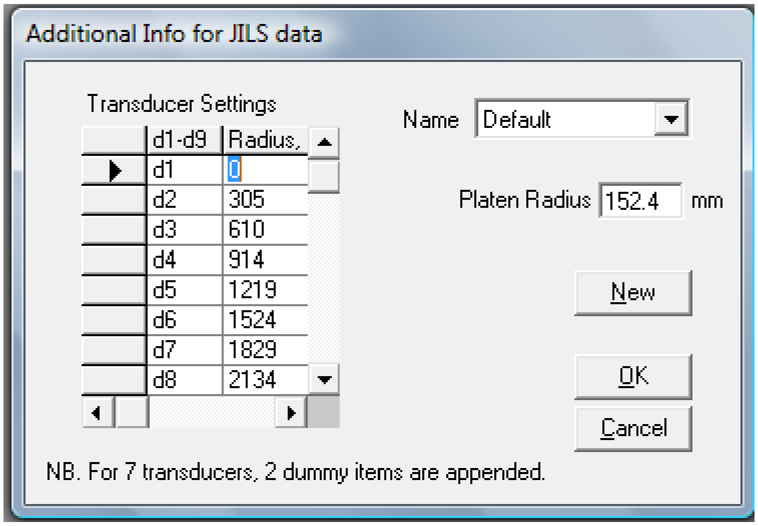

## Set Datafile Extension

The user can define the data file extension. This is useful in that non-needed data files can be filtered out by careful selection of datafile extensions.




## Settings for JILS data

In the JILS data file format various parameters (transducer spacing and load-plate diameter) are not defined. These must be defined in this dialog box to enable back-calculation.


Additional Info for JILS data
Transducer Settings
d1-d9
Radius,
d1
0
d2
305
d3
610
d4
914
d5
1219
d6
1524
d7
1829
d8
2134
Name
Default
Platen Radius
152.4
mm
New
OK
Cancel
NB. For 7 transducers, 2 dummy items are appended.

# References

Thompson, M.R. "ILLI-PAVE Based NDT Analysis Procedures', Nondestructive testing of pavements and Backcalculation of modulii, ASTM STP 1026, 1989.

Thompson, M.R. Discussion, 5th Int. Conf. Struct. Design of Asphalt Pavements, Vol.II pp.293-294, 1982.

Press, W.H., Flannery, B.P., Teukolsky, S.A. & Vetterling, W.T. 'Numerical Recipes", Cambridge Univ. Press, Cambridge, 1986.